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Abstract

Many governments aim to give disadvantaged firms an equal opportunity to compete for government contracts and will use diversity in awards as a measure of success. I show theoretically that, when *ex ante* identical disadvantaged firms differ in an identifiable but irrelevant characteristic, diversity in awards may not translate into equity in opportunity—as buyers may discriminate within the disadvantaged group. Subcontracting data on Disadvantaged Business Enterprises in New Mexico show that inequities can arise in practice.

Keywords: Affirmative action, statistical discrimination, government contracting.

JEL Codes: D63, H57, J71, L24.

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1 Introduction

Government spending on procurement contracting is traditionally large, amounting to 12 percent of Organization for Economic Cooperation and Development (OECD) GDP annually.[1]As a means of creating a "level playing field" on which firms from disadvantaged backgrounds have an equal opportunity to compete, many contracting agencies have established diversity goals. Examples include Australia's three percent goal for the number of contracts awarded to indigenous firms and the U.S.'s five percent goal of total contract dollars to be awarded to socially and economically disadvantaged businesses. In reaching these goals, agencies may rely on affirmative action policies, which favor awarding contracts to those firms considered disadvantaged.

Despite these goals and policies, disadvantaged contractors often cite discrimination as a barrier to winning public contracts. However, this discrimination tends to be in the form of network access, in the sense that government agencies and prime contractors favor working with known or established firms. Indeed, a meta-analysis of disparity studies commissioned by the U.S. Department of Commerce, which covered 31 different states and every major Census region, found these network access issues were a barrier to minority- and women-owned business enterprises in nearly every study (Premier Quantitative Consulting, 2016). As a result, some of these enterprises reported facing stereotyping and higher standards when competing for government contracts.

More generally, if agencies and prime contractors can discriminate on a characteristic outside of a firm's disadvantaged status, highly inequitable outcomes can arise which might appear diverse. In this case, a disadvantaged firm might be likely to win a contract, but only if it has the characteristic of being established or well-known. An equally capable but unknown disadvantaged firm may not have a chance to win, and affirmative action policies for disadvantaged firms as a whole may not reliably correct these inequities.

This paper explores these issues theoretically by embedding a statistical discrimination framework within a contracting model. In the model, a buyer—who can be either a government contracting officer or a non-disadvantaged prime contractor—can contract (or subcontract) with one of two disadvantaged firms or an outside option (representing an advantaged firm or the potential to do the work in-house). The disadvantaged firms must incur a randomly distributed cost to become qualified to perform the required work and differ only in a label that has no bearing on the distribution of those costs. This label can be established versus unestablished, male-owned versus female-owned, or any other distinction that can occur within the disadvantaged group. The buyer receives a noisy signal from each disadvantaged firm of whether it is qualified and uses those signals to choose a firm (or the outside option).

[1]See the OECD's Government at a Glance report, available at `https://www.oecd-ilibrary.org/governance/government-at-a-glance_22214399`.

I show that many different outcomes can arise in equilibrium—some intended and some likely unintended. On the one hand, the buyer can ignore the label entirely, treating all disadvantaged firms equally. In this case, diversity in contract awards between disadvantaged and non-disadvantaged firms coincides with equity for firms within the disadvantaged group. On the other hand, buyers can use labels exclusively, meaning that the disadvantaged firm with the unfavorable label is never utilized. In this case, there can be diversity in contract awards but stereotyping, higher standards, and inequities within the disadvantaged group—contradicting the idea of creating a level playing field.

Next, I consider several interventions. When governments implement a U.S.-inspired affirmative action program, there are equilibria where a disadvantaged firm is more likely to win. However, buyers can still use labels when selecting firms, so these equilibria can be inequitable. A more targeted subsidy policy for disadvantaged firms with the unfavorable label may still fail to address these inequities, as the unfavorable label can carry enough weight in a buyer's selection decision to offset any benefits from issuing a subsidy. In an environment where governments mandate equity by randomly awarding contracts through pre-specified win probabilities, a unique equilibrium arises in which firms never become qualified. The intuition here is that competition for the contract generates incentives to become qualified, and by assigning contracts equitably through a random process, governments remove that crucial incentive. These results highlight the complexity of addressing inequities through interventions when buyers can discriminate on labels.

Subcontracting data from highway construction contracts in New Mexico further motivate the model's equilibria. For one of the five most common types of work, I find that contract awards are unconcentrated in the disadvantaged group, consistent with an equilibrium where buyers largely ignore labels. For the other four work types, I find that contract awards in the disadvantaged group are highly concentrated, implying that few disadvantaged firms win most of the contracts. These results are consistent with an equilibrium where buyers discriminate using labels and suggest that inequities can appear in real-life contracting settings.

This paper fits in a rich literature on equilibrium models of statistical discrimination, beginning with Arrow (1973).[2]A key feature in his model is that differences between *ex ante* similar groups can arise in equilibrium due to self-fulfilling beliefs (or stereotypes). Coate and Loury (1993) formalized that idea in their model of employer task assignment, finding that affirmative action may fail to correct negative group stereotypes. In both models, discrimination arises as a coordination failure, in the sense that the discriminated group fails to coordinate on the other group's equilibrium.

There have been many other influential contributions to this framework since then. Moro and Norman (2004) study statistical discrimination within a general equilibrium labor market

[2]For an in-depth survey of statistical discrimination models, see Fang and Moro (2011).

model, where inter-group interactions generate a form of discrimination that does not have the usual coordination failure explanation. Fang and Norman (2006) consider a model with a public and private sector and show that the discriminated group can be economically more successful even when excluded from the more lucrative sector. Fryer (2007) investigates a dynamic model of discrimination and provides sufficient conditions for belief flipping, while Craig and Fryer (2018) allow both workers and firms to discriminate in a two-sided model of statistical discrimination. Guha and Roy Chowdhury (2022) consider a model where firms can discriminate on both class and race, allowing for discrimination within a group receiving affirmative action—similar to my setup.

This paper contributes to the statistical discrimination literature by extending it to a contracting setting. Like labor markets, government contracting is an area that has been subject to affirmative action policies for decades as a means of correcting past and present forms of discrimination. These settings differ in that it is competition between contractors that determines incentives to become qualified in government contracting, as opposed to the potential earnings differences that drive incentives in a labor market. This between-contractor competition generates an interaction that rules out coordination failure as an explanation for discrimination and creates several challenges when implementing corrective policies. This paper also differs from much of the statistical discrimination literature in its focus on discrimination within the disadvantaged group and the potential challenges governments face in attempting to create a truly level playing field.

Fryer and Loury (2005) explore affirmative action in a winner-take-all market, which relates to the markets in which contracts are awarded. A key difference between this paper and theirs is that they allow agents to signal their type through an observable effort choice, whereas signals in this paper correlate to an unobserved investment to become qualified. In many contracting settings, buyers may not be able to observe effort directly, leading to an informational friction that can generate statistical discrimination. I build on their work by allowing this friction.

This paper also relates to an empirical literature on affirmative action in government contracting. Rosa (2020), De Silva et al. (2012), and Marion (2009) study subcontracting requirements favoring disadvantaged subcontractors in construction, finding that the requirements increase participation with either negligible or moderately increased costs to the government. Athey et al. (2013) consider set-aside and subsidy programs in timber contracting, finding that both policies increase small-firm participation with various consequences for efficiency and revenue. This paper shows that even if these programs increase diversity in contract awards, discrimination on an irrelevant characteristic within the disadvantaged group may render these awards inequitable.

The remainder of this paper proceeds as follows. Section 2 presents motivating evidence using data from New Mexico. Section 3 develops the theoretical model where buyers can discriminate among disadvantaged contractors, and Section 4 discusses the model's limitations and extensions. Section 5 concludes.

2 Motivating Evidence from New Mexico

This section explores an empirical setting where disadvantaged firms compete for government contracts. The purpose of this section is to demonstrate the possibility of diverse but inequitable contract awards in a real-life case, further motivating the theoretical analysis.

I consider data from construction contracts issued by the state of New Mexico between January 2008 and January 2015 for the maintenance, construction, and reconstruction of transportation systems. The New Mexico Department of Transportation (NMDOT) is the state department responsible for issuing these contracts, and in compliance with federal contracting requirements, must ensure that a certain share of contracts is awarded to firms designated as disadvantaged. In New Mexico, the disadvantaged designation generally requires that a firm be small and owned and controlled by ethnic minorities or women—with ownership meaning a 51 percent share in the firm and control meaning substantive decision-making power.

At the time, the NMDOT had a Disadvantaged Business Enterprise (DBE) Program that used race-conscious measures on some contracts to achieve their distributional objectives. These measures took the form of DBE goals which specify, on a contract-by-contract basis, a share of the award amount that must be given to DBE subcontractors. For example, a five percent share on a \$100 contract would require \$5 to be awarded to DBE subcontractors. These goals were not quotas: prime contractors could miss the goal and still be awarded the contract, provided they show sufficient good faith efforts to use DBE subcontractors.

My focus is on the distribution of subcontracts to and within the disadvantaged group of subcontractors. To that end, I use data from New Mexico's SHARE system in constructing the relevant contract- and subcontract-level data. The SHARE data come from New Mexico's accounting system and track transactions between New Mexico and its outside vendors. The data also contain information on whether the vendor is a subcontractor or prime contractor, whether the vendor is a DBE, and what type of work that vendor performed. Because DBE prime contractors must always perform more work than would be required by a goal to qualify as a prime—and are thus not bound by affirmative action requirements—I do not include them in the analysis.

I combine the SHARE information with data from the NMDOT's DBE directory. This directory lists all certified DBEs and a statement of their capabilities. I use the work listed in the capabilities statements as a measure of which DBEs are available. For example, a capabilities statement may list "trucking" as a capability, so I would consider that firm available for trucking-related work. In total, I observe 560 construction contracts, with 280 of them having DBE goals.

There are a total of 1126 subcontracts.

Table 1 summarizes the data at the contract level. Half of the contracts have a race-conscious DBE goal, and the average DBE share is about 7 percent. Although not shown in the table, the total DBE share in all work is 7.84 percent—which is within the historical 7 to 9 percent departmental goal, even excluding DBE prime contractors.[3]

Table 1: Summary Statistics for NMDOT Construction Contracts

	Mean	Stdev	Min	Median	Max	Obs
All projects						
Contract Value ($ M)	3.818	6.742	0.005	1.589	75.000	560
Has goal	0.500	0.500	0.000	0.500	1.000	560
DBE share (%)	7.099	8.467	0.000	4.751	47.293	560
Projects with goal						
Contract Value ($ M)	4.786	7.878	0.093	2.715	75.000	280
DBE share (%)	10.471	8.833	0.000	7.646	47.293	280
Projects without goal						
Contract Value ($ M)	2.850	5.209	0.005	1.002	41.832	280
DBE share (%)	3.726	6.542	0.000	0.927	45.579	280

Note: Contract value is the winning bid for the contract. Has goal is an indicator of whether there is a DBE goal on a contract. DBE share is the percentage share of a project's dollar value awarded to DBE firms.

Given the apparent diversity in subcontract awards, there is a question as to whether those awards are shared equitably *within* the group of disadvantaged firms. To explore this question further, I consider various concentration measures often utilized by antitrust authorities.

I focus on concentration within the five most common work types: trucking, painting, signing, fencing, and concrete. I use two different concentration indices. The first index is the three-firm concentration ratio, which is the market share of the three largest DBE subcontractors. The second index is the Herfindahl-Hirschman Index (HHI), which is defined as follows:

$$HHI = s_1^2 + s_2^2 + \cdots + s_n^2,$$

where $s_i \in [0, 100]$ is the share of DBE contract dollars awarded to firm i on a given work type. A high HHI or three-firm concentration ratio indicates more concentration, meaning that a small number of disadvantaged firms win most of the contracts.

Table 2 summarizes the concentration measures, firm availability, and the number of subcontracts in the New Mexico data. Of the five work types listed, trucking is the most popular

[3] At the time, the NMDOT determined DBE goals tri-annually based on aggregate DBE participation (NMDOT, 2012), which is why the goal ranges from 7 to 9 percent. For a more detailed description of the NMDOT's goal-setting and qualification procedures, see Rosa (2020).

subcontract and the least concentrated. The other four work types are relatively more concentrated, with the top three DBE firms winning 70.1 to 98.6 percent of all awards.

Table 2: Concentration Measures

	3-firm CR			HHI			Firms	Number of
	All	Goal	No goal	All	Goal	No goal	listed	subcontracts
Trucking	47.3	43.7	61.7	1138.9	1076.7	1634.3	29	184
Painting	98.6	98.1	100.0	8529.2	8281.6	9201.5	14	160
Signing	93.3	91.8	100.0	6853.0	6439.3	8387.4	10	109
Fencing	88.1	89.6	98.0	4077.2	4042.1	4854.4	10	93
Concrete	70.1	68.6	95.0	2777.7	2523.2	6967.4	37	74

Similarly, HHIs are high for all work types, excluding trucking. To put these numbers into perspective, I note that the 2010 Horizontal Merger Guidelines classify markets with an HHI above 2500 as highly concentrated, below 1500 as unconcentrated, and between 1500 and 2500 as moderately concentrated (DOJ, 2010). Although all trucking would be considered unconcentrated under these guidelines, subcontract awards are highly concentrated for every other work type, irrespective of whether there is a DBE subcontracting requirement. These descriptive results suggest that there can be inequities within the disadvantaged group in otherwise diverse contract awards.[4]

3 A Model of Disadvantaged Contractor Selection

There are many potential explanations for the concentration observed in the New Mexico data. For instance, disadvantaged firms may differ in location, capacity, experience, or other features. Past business relationships may also play a role in shaping subcontractor choices, either through reputations or relational contracting. Given that contracting data are rarely comprehensive enough to explore all of these issues separately, doing so here would be particularly challenging.

As a result, I ask a more fundamental question in this section: if the government could erase all of these *ex ante* advantages between disadvantaged firms, could it guarantee that diversity in contract awards implies equity within the disadvantaged group? In answering this question, I develop a stylized model that assumes contractors are equally situated and that agents are concerned about whether disadvantaged firms are qualified. These qualification concerns are significant in real-world contracting and often outweigh price considerations. I discuss alterations to this framework later in Section 4.

[4] I note that there are reasons why concentration may be high without inequity being the cause. For example, a disadvantaged firm may win a large subcontract relative to the other firms, so awards would appear to be concentrated on that firm. Also, there might be a small number of firms capable of doing a particular type of work, leading to a high concentration mechanically. In the New Mexico data, firms with a large share of subcontract dollars also win more contracts, and the work types appear to have many available firms.

3.1 Environment

Consider an environment where a buyer aims to procure goods or a service. This buyer can be either a contracting officer working on behalf of a government agency or a prime contractor selecting a subcontractor to perform part of a larger contract. The buyer can choose between two firms designated as disadvantaged or an outside option, where the outside option can be an advantaged firm or the option to perform the work in-house.

Each disadvantaged firm belongs to one of two identifiable groups. One disadvantaged firm belongs to an "established" or known group of firms, which I denote by an E subscript. The other disadvantaged firm is part of an "unestablished" or unknown group of firms, which I subscript by U. Aside from these labels, disadvantaged firms will be similar *ex ante*.

Because firms may not be familiar with the full extent of the work requested by the buyer, I assume firms start as unqualified but can become qualified through a costly investment. The cost of this investment is c and is distributed randomly according to a continuous and differentiable distribution function, G. In practice, this cost may represent time spent understanding the work or the cost of employing additional laborers.

Buyers that hire an unqualified firm often have to spend more time helping it complete the work and may face costly delays when executing the contract. These costs are in addition to the price the buyer would pay to the firm when recruiting it. As such, I assume a buyer opting to use a disadvantaged firm pays an exogenous price, p,[5]to the firm and incurs an additional cost that depends on the firm's qualification. The additional cost of using a qualified firm is x_q, while the additional cost of using an unqualified firm is x_u, where $x_q < x_u$. The total cost of using a disadvantaged firm is then $p + x_q$ if that firm is qualified and $p + x_u$ if unqualified.

The outside option's total cost is z, the realization of a random variable with distribution function F_z and support $[\underline{z}, \overline{z}]$. To rule out situations where buyer's prefer or reject disadvantaged firms due to *ex ante* differences in total costs, I assume $p + x_q \leq z < p + x_u$ for all $z \in [\underline{z}, \overline{z}]$. The buyer's value for completing the work is v, where $v \geq p + x_u$. This assumption rules out individual rationality issues.

The buyer does not observe whether a disadvantaged firm is qualified. Instead, the buyer observes a noisy signal, $\theta \in [0, 1]$. This signal can come from interviewing the disadvantaged firms directly and reading summaries of the type of work or services the firms can provide. The signal's distribution is continuous, differentiable, and depends on whether the firm is qualified: a qualified disadvantaged firm's signal follows the distribution function F_q, and an unqualified disadvantaged firm's signal follows the distribution function F_u—with densities f_q and f_u, respectively. I assume the densities satisfy the monotone likelihood ratio property (MLRP)

[5]Many goods and services have established catalog or market prices that buyers will use as a basis for pricing (see Federal Acquisition Regulation 15.402). Indeed, the construction industry has Blue Book prices, which are prices considered "fair and reasonable" for various work on new construction, remodeling, and repairs.

outlined in Assumption 1.

Assumption 1 (MLRP)**.** *The fraction $f_q(\theta)/f_u(\theta)$ is strictly increasing and continuous in θ for all $\theta \in [0, 1]$.*

This assumption means a buyer that receives a high signal from a disadvantaged firm will be more inclined to believe the firm is qualified. Furthermore, note that both established and unestablished disadvantaged firms have the same signal distribution conditional on investment, so a disadvantaged firm's group has no influence on the signal's accuracy.

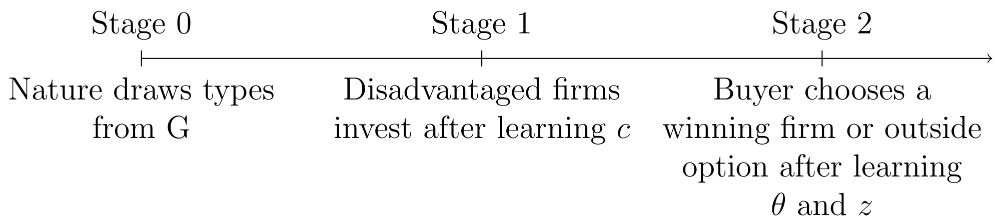


Figure 1: Timing of Events

The game proceeds as follows. Nature begins by drawing types from G for each of the disadvantaged firms. Next, disadvantaged firms decide whether to invest and become qualified. The buyer then decides whether to use one of the disadvantaged firms or the outside option after learning each firms' signal and the outside-option cost. Figure 1 outlines the timing of events. I aim to characterize the Bayesian Nash equilibria of this contractor-selection game.

3.2 The Buyer's Choice

The buyer chooses between using a disadvantaged firm or the outside option. Because the buyer cannot observe the disadvantaged firms' investment decisions directly, the buyer must infer them from the signals. Let $\pi_j \in [0, 1]$ be the common prior belief that disadvantaged firm $j \in \{E, U\}$ is qualified. After receiving signal θ_j, the buyer's posterior belief is

$$P(\theta_j; \pi_j) = \frac{\pi_j f_q(\theta_j)}{\pi_j f_q(\theta_j) + (1 - \pi_j) f_u(\theta_j)},$$

the result of Bayesian updating. Thus, a high signal or prior is beneficial to a disadvantaged firm in convincing the buyer that it is qualified.

Given the buyer's posterior beliefs, the expected cost, EC, of using disadvantaged firm j is

$$EC(\theta_j; \pi_j) = P(\theta_j; \pi_j)[p + x_q] + (1 - P(\theta_j; \pi_j))[p + x_u].$$

The buyer's interim payoff function is then

$$\Pi(v, z, \theta_E, \theta_U, \pi_E, \pi_U) = \begin{cases} v - EC(\theta_j; \pi_j) & \text{if the firm from group } j \in \{E, U\} \text{ is chosen.} \\ v - z & \text{if the outside option is chosen.} \end{cases}$$

Because $EC(\theta_j; \pi_j)$ is strictly decreasing in $P(\theta_j; \pi_j)$, a buyer's strategy is to select the disadvantaged firm with the highest posterior, provided that $EC(\theta_j; \pi_j) \leq z$ for the high-posterior firm.

The final component of the buyer's problem is how they break ties. I assume the buyer breaks ties between disadvantaged firms of different groups in favor of the established firm. The buyer breaks ties between a disadvantaged firm and the outside option in favor of the disadvantaged firm. Three-way ties go to the established disadvantaged firm.

3.3 The Investment Decision

Now consider a disadvantaged firm's investment decision. Intuitively, a disadvantaged firm will invest if its payoff from doing so exceeds its payoff from not investing. Recall that the buyer pays the firm p if it wins, but an investing firm must pay cost c irrespective of whether it wins or loses. Thus, the payoff from investing for firm j is

$$\Pr_j(\text{win} \mid \text{invest})p - c,$$

while the payoff from not investing is

$$\Pr_j(\text{win} \mid \text{no invest})p,$$

where $\Pr_j(\text{win} \mid \text{invest})$ and $\Pr_j(\text{win} \mid \text{no invest})$ denote the probability of winning given investment and the probability of winning without investment, respectively.

Because signals and outside-option draws are independent, one can break down the probability of winning given investment further as

$$\Pr_j(\text{win} \mid \text{invest}) = \Pr_j(\text{beat competitor} \mid \text{invest}) \times \Pr_j(\text{beat outside opt} \mid \text{invest}),$$

where $\Pr_j(\text{beat competitor} \mid \text{invest})$ is the probability that disadvantaged firm j beats its competitor given investment, and $\Pr_j(\text{beat outside opt} \mid \text{invest})$ is the probability that the firm beats the outside option if it invests.

To beat the outside option, a firm must transmit a sufficiently high signal so that $EC(\theta_j; \pi_j) \leq z$. Noting that $x_q - x_u < 0$, one can rewrite $EC(\theta_j; \pi_j) \leq z$ in terms of the posterior:

$P(\theta_j; \pi_j) \geq \frac{z-p-x_u}{x_q-x_u}$. Hence,

$$\Pr_j(\text{beat outside opt} \mid \text{invest}) = \int_{\underline{z}}^{\overline{z}} \int_0^1 1\left\{P(\tilde{\theta}; \pi_j) \geq \frac{\tilde{z}-p-x_u}{x_q-x_u}\right\} f_q(\tilde{\theta}) f_z(\tilde{z}) d\tilde{\theta} d\tilde{z}.$$

To beat its rival, a firm's signal must generate a posterior that exceeds its rival's. For the unestablished firm,

$$\begin{aligned}\Pr_U(\text{beat competitor} \mid \text{invest}) = \;& \pi_E \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_q(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\ +\;& (1-\pi_E) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_u(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta},\end{aligned}$$

and for the established firm,

$$\begin{aligned}\Pr_E(\text{beat competitor} \mid \text{invest}) = \;& \pi_U \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_q(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\ +\;& (1-\pi_U) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_u(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta}.\end{aligned}$$

Observe that the tie-breaking rule results in group-specific expressions for the probability of winning because the established firm can win in the event of a tie.

The no-investment formulation is similar, with the difference being that the firm draws its signal from the unqualified distribution. Thus,

$$\Pr_j(\text{win} \mid \text{no invest}) = \Pr_j(\text{beat competitor} \mid \text{no invest}) \times \Pr_j(\text{beat outside opt} \mid \text{no invest}),$$

where

$$\Pr_j(\text{beat outside opt} \mid \text{no invest}) = \int_{\underline{z}}^{\overline{z}} \int_0^1 1\left\{P(\tilde{\theta}; \pi_j) \geq \frac{\tilde{z}-p-x_u}{x_q-x_u}\right\} f_u(\tilde{\theta}) f_z(\tilde{z}) d\tilde{\theta} d\tilde{z},$$

$$\begin{aligned}\Pr_U(\text{beat competitor} \mid \text{no invest}) = \;& \pi_E \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_q(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\ +\;& (1-\pi_E) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_u(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta},\end{aligned}$$

and

$$\begin{aligned}\Pr_E(\text{beat competitor} \mid \text{no invest}) = \;& \pi_U \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_q(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\ +\;& (1-\pi_U) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_u(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta}.\end{aligned}$$

Given these expressions, firm j invests if

$$\Pr_j(\text{win} \mid \text{no invest}) p \leq \Pr_j(\text{win} \mid \text{invest}) p - c,$$

which can be rewritten as

$$c \leq \underbrace{\left[\Pr_j(\text{win} \mid \text{invest}) - \Pr_j(\text{win} \mid \text{no invest})\right] p}_{I_j(\pi_j, \pi_{-j})},$$

where π_{-j} is the rival prior.

Following notation from Moro and Norman (2004) and Fang and Moro (2011), the term $I_j(\pi_j, \pi_{-j})$ denotes a disadvantaged firm's benefit from investment, which I will refer to as its investment incentive. In contrast to many of the labor-market models of discrimination, where incentives are based on either the probability of meeting a group-specific threshold (as in Coate and Loury (1993) and Guha and Roy Chowdhury (2022)) or differences in potential earnings gains (as in Moro and Norman (2004)), the incentives to invest here depend on how much investment increases the probability of beating the rival firm and outside option. Put differently, the more investment increases the probability of winning relative to not investing, the more likely a disadvantaged firm is to invest.

Also, note that investment incentives between disadvantaged firms are connected; winning requires that a firm beat its competitor, which depends on a buyer's prior on both groups of firms. As a result, improving, say, the unestablished firm's prior will adversely affect the established group, ruling out coordination failure as an explanation for discrimination.

3.4 Equilibrium

In a Bayesian Nash equilibrium, prior beliefs are self-fulfilling and must align with actual investment probabilities. The idea here is that if the buyer has a low prior that a certain group of firms is qualified, then they are less likely to award a firm from that group a contract. That firm is then less likely to invest, justifying the buyer's pessimistic prior.

In the model, the probability that a firm invests is the probability of drawing an investment cost less than $I_j(\pi_j, \pi_{-j})$, or $G(I_j(\pi_j, \pi_{-j}))$. In equilibrium, priors match this probability, leading to the following definition.

Definition 1 (Equilibrium)**.** *An equilibrium is a set of beliefs* (π^*_E, π^*_U) *satisfying for each* $j \in \{E, U\}$

$$\pi^*_j = G(I_j(\pi^*_j, \pi^*_{-j})). \tag{1}$$

Depending on the investment cost, signal, and outside option distributions, there can be many potential solutions to (1). Some of these solutions have equivalent priors across groups, so the buyer treats established and unestablished firms equally. In other solutions, group priors are distinct, leading to discrimination within disadvantaged firms.

3.4.1 Symmetric Equilibria

I begin by analyzing symmetric equilibria—the solutions to (1) where disadvantaged firms from different groups have the same investment probabilities (and priors). In these equilibria, the buyer ignores the "established" and "unestablished" labels and treats both firms equitably.[6]

Because symmetry requires the buyer to have the same priors across groups, one can simplify the model's notation. Specifically, symmetry implies $\pi_j = \pi_{-j} = \pi$, so that $I_j(\pi_j, \pi_{-j}) = I_j(\pi, \pi) = I_j(\pi)$. Symmetric equilibria are then solutions to $\pi^* = G(I_j(\pi^*))$.

As long as investment costs are non-negative, one can always construct an equilibrium where disadvantaged firms never invest and are thus never utilized. In these solutions, a buyer with a prior that no disadvantaged firms invest (or $\pi = 0$) will have the same no-investment posterior ($P(\theta_j; 0) = 0$) irrespective of the signals. Because signals do not change priors, they do not affect the probability of winning, and there is no incentive for a disadvantaged firm to invest in the hopes of attaining a higher one. Hence, $I_j(0) = 0$. Non-negative investment costs together with the assumptions on G imply that $G(0) = 0$, so $\pi^* = 0 = G(I_j(0))$ is an equilibrium.

Symmetric equilibria with positive investment probabilities can also exist with strong enough incentives, but one must take care in analyzing investment incentives when priors are near one. When a buyer's prior is that every disadvantaged firm invests ($\pi = 1$), their posterior will not change for any signal ($P(\theta_j; 1) = 1$ for every θ_j). As was true with the no-investment prior, there is no incentive for disadvantaged firms to invest when signals do not affect posteriors, implying $I_j(1) = 0$. However, investment incentives can be positive arbitrarily close to $\pi = 1$, leading to a discontinuity.

Intuitively, for a symmetric prior arbitrarily close to one, the buyer views both disadvantaged firms as equal and can be persuaded by a high signal to choose one firm over the other. Therefore, there can still be a positive investment incentive. This discontinuity will not appear elsewhere—as the outside option forces incentives to zero near $\pi = 0$, and the buyer will be willing to overlook a firm's poor signal if its group's prior is high enough relative to the competing firm.

Given this potential discontinuity, one sufficient condition for a symmetric equilibrium with positive investment is that the support of G is large enough relative to the investment incentives. This condition rules out cases where incentives are so high that every firm would invest, but every firm investing would mean there is no incentive in the first place. Formally, I assume $\lim_{\pi \nearrow 1} G(I_j(\pi)) < 1$, where $\lim_{\pi \nearrow 1}$ denotes the limit as π approaches one. The other condition for a positive-investment equilibrium is that incentives are high enough for $G(I_j(\pi))$ to exceed π at some π. When these conditions hold, one can show that there is a symmetric equilibrium with positive investment probabilities.

Figure 2 plots several potential investment incentive functions and their equilibria. Here,

[6] These equilibria would also arise if the disadvantaged firms were from the same group.

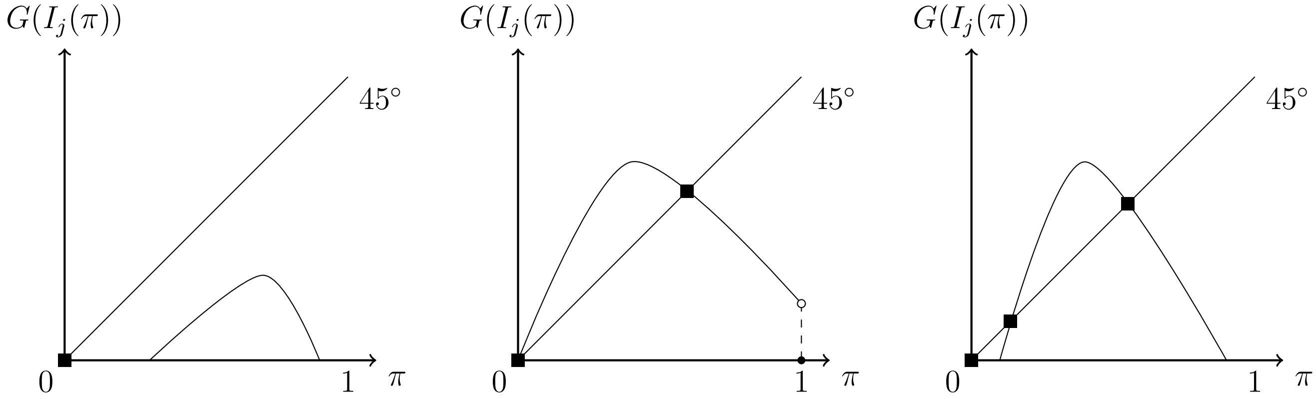


Figure 2: Example Symmetric Equilibria

an equilibrium is a fixed point occurring when $G(I_j(\pi))$ crosses the 45-degree line. In the left panel, incentives are so low that the no-investment equilibrium is unique. In the middle and right panels, the no-investment equilibrium still exists, but investment incentives are high enough at some prior values to generate equilibria with positive investment. The middle panel has a discontinuity at one, but incentives approaching one are low enough for the lines to cross at a positive π.

Proposition 1 formalizes these ideas; all proofs are in Appendix A.

Proposition 1. *If $G(c)$ is continuous and satisfies $G(0) = 0$, then a symmetric equilibrium exists. Furthermore, if $G^{-1}(\pi) < I_j(\pi)$ for some $\pi \in (0,1)$ and $\lim_{\pi \nearrow 1} G(I_j(\pi)) < 1$, then a symmetric equilibrium with $\pi^* > 0$ exists.*

This proposition demonstrates the potential for an equitable distribution of contracts across disadvantaged firms absent intervention. However, there is always the possibility of a no-investment equilibrium, meaning buyers can completely shut disadvantaged firms out of contracting.

3.4.2 Asymmetric Equilibria

I now consider the possibility of asymmetric equilibria, which are solutions to (1) where $\pi_E \neq \pi_U$. In these equilibria, the buyer uses labels when selecting contractors, even though they have no bearing on the investment-cost distribution. These equilibria are discriminatory; buyers (correctly) believe the firm with the lower prior is less likely to be qualified. These beliefs then generate a double standard: because posterior beliefs are increasing in priors, buyers may turn down the low-prior firm despite having a higher signal than the high-prior firm.

As noted by Moro and Norman (2004), characterizing a set of conditions for all asymmetric equilibria is challenging in models where investment incentives across groups interact, even in tightly parameterized models. They find a general result by investigating distributions where an employer assigns all agents from one group to a simple task in their job-assignment model. I take a similar approach here by exploring equilibria where the discriminated group is never awarded the contract. I call such equilibria *asymmetric equilibria with exclusion*, which I define as follows.

Definition 2 (Asymmetric Equilibria with Exclusion)**.** *In asymmetric equilibria with exclusion, $\pi_j^* > 0$ for the firm in group $j \in \{E, U\}$, while $\pi_{-j}^* = 0$ for the other firm.*

When they exist, asymmetric equilibria with exclusion combine the intuition from the no-investment and positive-investment symmetric equilibria. If $\pi_U = 0$, the unestablished firm has no incentive to invest because the buyer will never believe it has invested. Meanwhile, the established firm will invest with some positive probability if incentives are strong enough. The difference here is that the established firm behaves as if it is competing against the outside option only because the unestablished firm can never win.

Proposition 2 states the existence criteria for asymmetric equilibria with exclusion formally.

Proposition 2. *If $G(c)$ is continuous and satisfies $G(0) = 0$ and $G^{-1}(\pi_j) < I_j(\pi_j, 0)$ for some $\pi_j \in (0, 1)$ and $j \in \{E, U\}$, then an asymmetric equilibrium with exclusion exists.*

In stark contrast to the symmetric case, this proposition shows that there can be highly inequitable situations where disadvantaged firms from the unestablished group can never win any contracts absent government intervention, even when they have the same investment costs as established firms. In what follows, I investigate a U.S.-inspired affirmative action intervention, the equilibria that result from its implementation, and whether it can prevent discriminatory equilibria within the disadvantaged group of firms.

3.5 Affirmative Action

In the U.S., affirmative action in public contracting began with President John F. Kennedy's Executive Order 10925, which obligated that government contracting agencies take "affirmative action" to ensure all qualified applicants on federal contracts are treated fairly regardless of race. Since then, affirmative action has expanded to cover other groups considered disadvantaged in contracting—such as women- and veteran-owned businesses—and is implemented through placement goals. These goals are targets considered attainable by "good faith efforts" to use disadvantaged firms in a marketplace, according to the Code of Federal Regulations (CFR, see 41 CFR 60-2.16).

Legally, placement goals cannot compel buyers to use unqualified firms and do not create set asides for disadvantaged groups. To account for these features in the model, I assume a contract subject to an affirmative action placement goal has an exogenous posterior cutoff, $\overline{P}$, set by an outside party. This outside-party assumption aligns with many real-world contracting settings, where the people responsible for setting and evaluating placement goals are often separate from buyers. For simplicity, I abstract away from explicitly modeling this party's incentives and assume buyers and firms take their decisions as given.[7]

A buyer that receives a signal from a disadvantaged firm low enough to generate a posterior below $\overline{P}$ would not need to use that disadvantaged firm. When posteriors for both disadvantaged firms are below this cutoff, the buyer can use the outside option, and if questioned by an outside official, a buyer rejecting both disadvantaged firms could presumably share the signals that led to their decision. This formulation captures the notion that a buyer can make a good faith effort to use disadvantaged firms (when the cutoff is low) but is not compelled to use disadvantaged firms they believe are likely unqualified. In practice, agencies often explicitly list good faith efforts as a valid reason for missing placement goals (see 49 CFR 26.53), and contracting officers who fail to receive competitive (or qualified) offers on a contract with a placement goal can seek outside offers.

To operationalize this cutoff, I make the following assumptions on its potential values.

Assumption 2 (Good Faith Effort Requirement)**.** *The good faith effort posterior cutoff satisfies* $\overline{P} > 0$ *and* $\overline{P} \leq \frac{\overline{z}-p-x_u}{x_q-x_u}$.

The requirement that $\overline{P} > 0$ prohibits set-aside contracts, as the buyer is never obligated to use a firm it (correctly) believes is unqualified with certainty. The requirement that $\overline{P} \leq \frac{\overline{z}-p-x_u}{x_q-x_u}$ means that the outside option is preferable to any disadvantaged firm below the cutoff and is derived by solving for the posterior that generates an expected cost higher than $\overline{z}$.[8]This condition ensures that the cutoff is low enough to affect behavior meaningfully and prevents buyers from wanting to use disadvantaged firms below the cutoff.

Affirmative action changes a disadvantaged firm's incentive to invest. Let $\hat{I}_j(\pi_j, \pi_{-j}; \overline{P})$ denote the incentive for disadvantaged firm j to invest given posterior cutoff $\overline{P}$. These incentives can be written as

$$\hat{I}_j(\pi_j, \pi_{-j}; \overline{P}) = \left[\Pr_j(\text{win} \mid \text{invest}, \overline{P}) - \Pr_j(\text{win} \mid \text{no invest}, \overline{P})\right] p,$$

where,

$$\begin{aligned}
\Pr_U(\text{win} \mid \text{invest}, \overline{P}) = \quad & \Big[\pi_E \textstyle\int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_q(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\
+ \quad & (1-\pi_E) \textstyle\int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_u(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta}\Big] \\
\times \quad & \textstyle\int_0^1 1\{P(\tilde{\theta}; \pi_U) \geq \overline{P}\} f_q(\tilde{\theta}) d\tilde{\theta},
\end{aligned}$$

[8]Specifically, $\overline{P}(p + x_q) + (1 - \overline{P})(p + x_u) \geq \overline{z}$.

$$\begin{aligned}
\Pr_E(\text{win} \mid \text{invest}, \overline{P}) = & \Big[\pi_U \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_q(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\
+ & (1 - \pi_U) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_u(\hat{\theta}) f_q(\tilde{\theta}) d\hat{\theta} d\tilde{\theta}\Big] \\
\times & \int_0^1 1\{P(\tilde{\theta}; \pi_E) \geq \overline{P}\} f_q(\tilde{\theta}) d\tilde{\theta},
\end{aligned}$$

$$\begin{aligned}
\Pr_U(\text{win} \mid \text{no invest}, \overline{P}) = & \Big[\pi_E \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_q(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\
+ & (1 - \pi_E) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_U) - P(\hat{\theta}; \pi_E) > 0\} f_u(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta}\Big] \\
\times & \int_0^1 1\{P(\tilde{\theta}; \pi_U) \geq \overline{P}\} f_u(\tilde{\theta}) d\tilde{\theta},
\end{aligned}$$

and

$$\begin{aligned}
\Pr_E(\text{win} \mid \text{no invest}, \overline{P}) = & \Big[\pi_U \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_q(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta} \\
+ & (1 - \pi_U) \int_0^1 \int_0^1 1\{P(\tilde{\theta}; \pi_E) - P(\hat{\theta}; \pi_U) \geq 0\} f_u(\hat{\theta}) f_u(\tilde{\theta}) d\hat{\theta} d\tilde{\theta}\Big] \\
\times & \int_0^1 1\{P(\tilde{\theta}; \pi_E) \geq \overline{P}\} f_u(\tilde{\theta}) d\tilde{\theta}.
\end{aligned}$$

The change in these expressions stems from the outside option no longer being relevant; instead, the buyer must hold a posterior belief that exceeds the good faith efforts threshold, which is inherently less competitive than the outside option.

Note that affirmative action can strengthen or weaken investment incentives for a particular belief configuration—in the sense that $I_j(\pi_j, \pi_{-j})$ can be greater than, less than, or equal to $\hat{I}_j(\pi_j, \pi_{-j}; \overline{P})$ depending on π_j, π_{-j}, $\overline{P}$, and the cost and signal distributions. For example, affirmative action strengthens investment incentives when investment generates a posterior that is more likely to surpass the threshold than the outside option, which happens when the outside option is inexpensive. Affirmative action could weaken incentives when priors are high, as a disadvantaged firm is more likely to get away with not investing and still make the good faith efforts requirement. In Section 3.6, I investigate a parameterized example illustrating these points further.

An equilibrium under affirmative action accounts for the posterior threshold and has the following definition.

Definition 3 (Equilibrium under Affirmative Action)**.** *An equilibrium under affirmative action is a set of beliefs* (π_E^*, π_U^*) *satisfying for each* $j \in \{E, U\}$

$$\pi_j^* = G(\hat{I}_j(\pi_j^*, \pi_{-j}^*; \overline{P})). \tag{2}$$

As was the case without affirmative action, there can be many solutions to (2). Next, I investigate these possibilities and provide conditions under which affirmative action generates equilibria with more disadvantaged investment than is possible absent intervention.

3.5.1 Symmetric Equilibria under Affirmative Action

First, consider the symmetric equilibria. Because affirmative action diminishes the competitiveness of the outside option, much of the intuition from the no-affirmative action case still applies. With high enough investment incentives, one can find an equilibrium with positive investment because those incentives will decline enough as a buyer's prior approaches one. Perhaps surprisingly though, the no-investment equilibrium can still exist, even for low values of $\overline{P}$. In this case, no disadvantaged firms ever become qualified, and the buyer chooses the outside option through good faith efforts. Corollary 1 describes when symmetric equilibria under affirmative action exist.

Corollary 1. *If $G(c)$ is continuous and satisfies $G(0) = 0$, then a symmetric equilibrium under affirmative action exists. Furthermore, if $G^{-1}(\pi) < \hat{I}_j(\pi; \overline{P})$ for some $\pi \in (0, 1)$ and $\lim_{\pi \nearrow 1} G(\hat{I}_j(\pi; \overline{P})) < 1$, then there exists a symmetric equilibrium under affirmative action where $\pi^* > 0$.*

Given the similarities in their existence, there is a question as to whether and how affirmative action affects investment probabilities. Recalling that affirmative action can either increase or decrease investment incentives depending on the buyer's prior, one can provide conditions under which affirmative action can lead to a higher equilibrium investment probability than is achievable without affirmative action. Proposition 3 provides those conditions.

Proposition 3. *Let $G(c)$ be continuous and satisfy $G(0) = 0$. Suppose π^* is the highest positive symmetric equilibrium investment probability, $I_j(\pi^*) < \hat{I}_j(\pi^*; \overline{P})$, $\lim_{\pi \nearrow 1} G(I_j(\pi)) < 1$, and $\lim_{\pi \nearrow 1} G(\hat{I}_j(\pi; \overline{P})) < 1$. Then there exists another investment probability π^{**} such that $\pi^{**} > \pi^*$ and π^{**} is an equilibrium investment probability under affirmative action.*

In words, when affirmative action makes investment more attractive at the highest equilibrium investment level without the policy, higher-cost disadvantaged firms will find it beneficial to invest. The buyer will rationally expect this development, so equilibrium investment will increase. Important here is that this upturn is shared equitably across established and unestablished firms. Thus, consistent with many of the intended goals agencies have when implementing such policies, affirmative action can increase investment for all groups of disadvantaged firms, leading them to win more contracts.

3.5.2 Asymmetric Equilibria under Affirmative Action

I now turn to a likely unintended set of equilibria that can occur under affirmative action. In these equilibria, affirmative action may cause disadvantaged firms to invest and win more, but only the established disadvantaged firm realizes these benefits. Corollary 2 states the conditions under which these equilibria exist.

Corollary 2. *If $G(c)$ is continuous and satisfies $G(0) = 0$ and $G^{-1}(\pi_j) < \hat{I}_j(\pi_j, 0; \overline{P})$ for some $\pi_j \in (0, 1)$, then an asymmetric equilibrium with exclusion under affirmative action exists.*

Corollary 2 says that, as long as investment costs are non-negative, there are equilibria where unestablished disadvantaged firms never invest and never win, even when there is affirmative action. The issue here is that the buyer may rationally expect that the unestablished firm will never invest and never use it while still fulfilling affirmative action requirements by using the established firm only.

Proposition 4 investigates how affirmative action can impact investment in these inequitable environments.

Proposition 4. *Let $G(c)$ be continuous and satisfy $G(0) = 0$. If π_j^* is the highest equilibrium investment probability for the non-excluded group in an asymmetric equilibrium with exclusion and $I_j(\pi_j^*, 0) < \hat{I}_j(\pi_j^*, 0; \overline{P})$, then there exists another investment probability π_j^{**} such that $\pi_j^{**} > \pi_j^*$ and π_j^{**} is an equilibrium investment probability in an asymmetric equilibrium with exclusion under affirmative action.*

This proposition highlights a potentially problematic equity issue related to affirmative action. Similar to Proposition 3, affirmative action can raise equilibrium investment levels and contract awards, but these increases would accrue to established firms only. In practice, this type of inequity can be missed when contracting agencies use the total share of contracts awarded to disadvantaged firms as an equity indicator. These results show that while affirmative action can increase the share of disadvantaged contract winners by inducing them to invest, there can still be discrimination against those disadvantaged firms labeled as unestablished.

3.6 A Parametric Illustration

Having established the existence of many possible equilibria and potential effects of affirmative action policies, I now turn to a concrete example with parameterized values. This example illustrates some of the earlier points on how affirmative action can affect investment incentives and facilitates a comparison of symmetric and asymmetric equilibria with exclusion.

Table 3 summarizes the parameterized values. In this example, I assume there is no extra cost to using a qualified disadvantaged firm ($x_q = 0$) but the additional cost of using an unqualified firm is equal to the market price (of $p = x_u = 0.5$). Furthermore, I assume a uniform investment and outside-option cost distribution—with a support of $[0, 1]$ and $[0.6, 0.8]$, respectively. Note that a buyer's expected cost of using a disadvantaged firm ranges from 0.5 to 1

Table 3: Parameter Values

Model Element	Meaning	Value
x_q	Additional cost of qualified firm	0
x_u	Additional cost of unqualified firm	0.5
p	Price to use firms	0.5
$\overline{P}$	Affirmative action posterior cutoff	0.25
G	Investment cost distribution	$\mathcal{U}(0,1)$
F_z	Outside option cost distribution	$\mathcal{U}(0.6,0.8)$
F_q	Qualified signal distribution	$\text{Beta}(3,1)$
F_u	Unqualified signal distribution	$\text{Beta}(1,3)$

Notes: $\mathcal{U}(a,b)$ corresponds to uniform distribution on the interval $[a,b]$. $\text{Beta}(a,b)$ denotes a beta distribution, with shape parameters a and b.

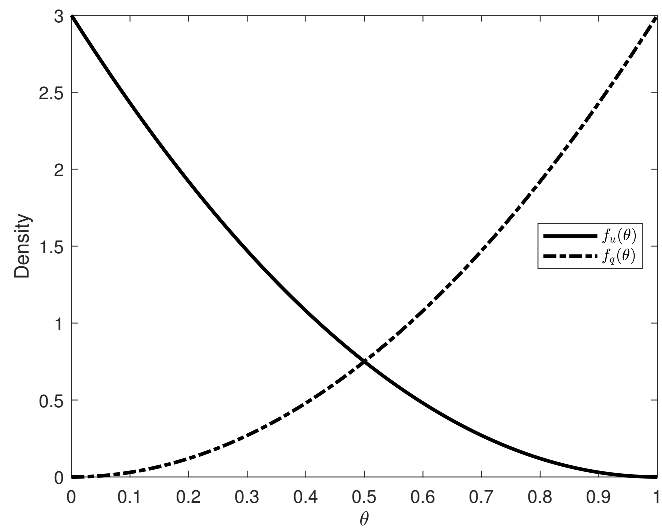
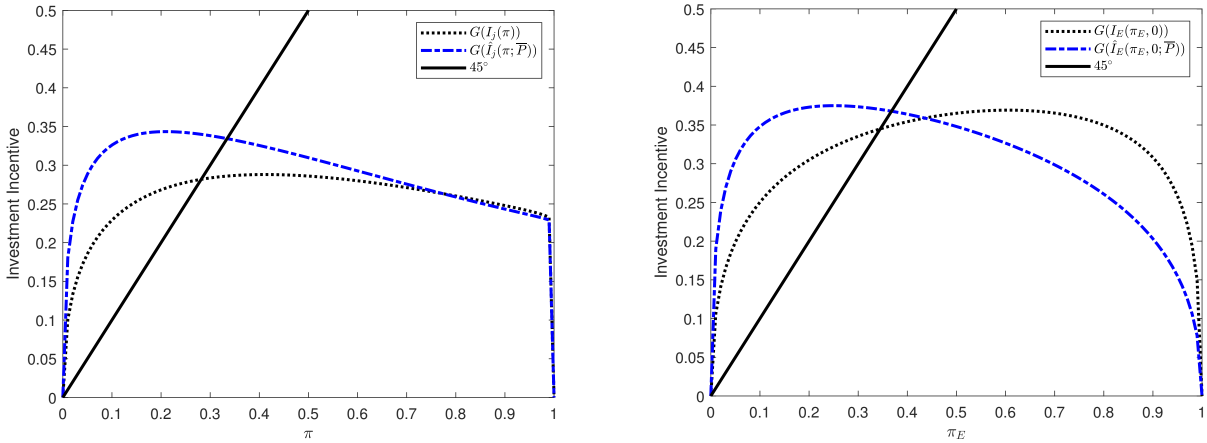


Figure 3: Signal Densities

depending on the buyer's posterior, so the range of expected costs contains the support of the outside-option distribution.

I assume signals follow a beta distribution with shape parameters that depend on whether a disadvantaged firm is qualified. Figure 3 plots the signal densities for the assumed parameter values. Because the qualified density is increasing in θ and the unqualified density is decreasing, it follows immediately that the monotone likelihood ratio property is satisfied. When it applies, I assume that the affirmative action posterior cutoff is $\overline{P} = 0.25$, which corresponds to an expected cost of 0.875.

Figure 4 plots the investment incentives that arise from the assumed parameterization. In constructing these figures, I solve for investment incentives on a uniform grid containing 10,000 different prior combinations. I obtain incentives for combinations outside the grid points via a modified Akima spline interpolation, which produces fewer undulations than a standard cubic



Symmetric Investment Incentives   Asymmetric Investment Incentives

Figure 4: Investment Incentives

spline. Each graph plots priors (on the x-axis) against investment incentives (on the y-axis). Note that investment incentives are equivalent to their CDF value, given the uniform assumption on investment costs.

The left panel in Figure 4 illustrates the investment incentives that arise in the symmetric case. Given the assumed parameter values, affirmative action here generally encourages investment, as incentives without affirmative action are lower than with affirmative action. As mentioned earlier, there is a discontinuity as incentives approach one; however, incentives approaching one are not high enough to cause every firm to invest. Proposition 3 then implies that equilibrium investment probabilities at the highest equilibrium should be even higher under affirmative action, which the rightward shift from the intersection between $G(I_j(\pi))$ and the 45-degree line to the intersection between $G(\hat{I}_j(\pi;\overline{P}))$ and the 45-degree line confirms.

The right panel of Figure 4 graphs investment incentives of the established firm in an asymmetric equilibrium with exclusion. This panel shows the potential for affirmative action to discourage investment, as incentives without affirmative action eventually exceed incentives with affirmative action for high priors. Intuitively, if the buyer were to have a high prior, the established firm is likely to meet the threshold without investment, leading to a low incentive. However, because the outside option is more competitive than the cutoff, the established firm is more inclined to invest when its prior is high. Nevertheless, incentives for investment are higher under affirmative action at the equilibrium prior, meaning that incentives meet the conditions outlined in Proposition 4. Thus, equilibrium investment is higher under affirmative action, but only the established firm ever reaps those benefits.

Table 4 calculates several equilibrium outcomes from the parameterized model. Because signals are reasonably accurate in this example, investment notably increases the probability of winning for disadvantaged firms in all cases. However, investment is costly enough that

Table 4: Equilibrium Outcomes from the Parametric Example

	Asymmetric equilibrium		Symmetric equilibrium
	Group E	Group U	
No affirmative action			
Prob. of investment	0.345	0.000	0.282
Prob. of winning given investment	0.745	0.000	0.579
Prob. of winning given no investment	0.055	0.000	0.016
Prob. of winning	0.293	0.000	0.174
Average expected cost	0.827	1.000	0.859
Affirmative action			
Prob. of investment	0.368	0.000	0.334
Prob. of winning given investment	0.920	0.000	0.727
Prob. of winning given no investment	0.184	0.000	0.058
Prob. of winning	0.455	0.000	0.282
Average expected cost	0.816	1.000	0.833

only about 28 to 37 percent of firms will invest in any equilibrium. As is implied by Figure 4, the probability of investment increases with affirmative action except for the unestablished firm when it is excluded. With increased investment probabilities, the average expected cost of using a disadvantaged firm declines, and the decreased competitiveness from the outside option increases overall win probabilities.[9]The caveat here is that the unestablished disadvantaged firm may experience none of these benefits should it find itself in the asymmetric equilibrium.

3.7 Other Government Interventions

The analysis so far has considered an affirmative action policy and its potential impacts on firms within the disadvantaged group. I have shown that these policies need not be equitable, as equilibria can arise where disadvantaged firms that have yet to establish themselves might face enough discrimination to be excluded from the market entirely. I now turn to an investigation of other equity-motivated interventions, whether they can rule out exclusionary equilibria, and their associated outcomes.

3.7.1 Proportional Subsidies

First, consider a proportional subsidy awarded to the unestablished disadvantaged firm by a government agency separate from the buyer. In the model, this subsidy takes on a value $\phi \in (0, 1)$, which reduces the investment cost of the unestablished firm from c to $c(1-\phi)$. I abstract away from reporting incentives by assuming the government can verify these costs.

[9]The probability of winning is a weighted average of the probability of winning with and without investment. That is, $\Pr(\text{win}) = \pi_j^* \Pr(\text{win} \mid \text{invest}) + (1-\pi_j^*)\Pr(\text{win} \mid \text{invest})$. The average expected cost is the *ex ante* expected cost of using a disadvantaged firm: $\pi_j^*(x_q + p) + (1-\pi_j^*)(x_u + p)$.

Proposition 5 shows that even a particularly generous subsidy program cannot rule out exclusionary equilibria.

Proposition 5. *Let $G(c)$ be continuous and satisfy $G(0) = 0$. If an asymmetric equilibrium with exclusion exists without a subsidy, then an asymmetric equilibrium with exclusion still exists for any proportional subsidy $\phi \in (0,1)$ to the excluded group.*

The idea here is that a highly subsidized unestablished firm may have the means to invest, but the buyer rationally believes such a firm would never want to—leading to a self-fulfilling spiral where the unestablished firm has no incentive to invest. Note that because the subsidy lowers the investment costs of unestablished firms, this result implies that an unestablished firm that is naturally less costly can still be excluded from the market. Thus, neither high subsidies nor less costly unestablished firms necessarily rule out discrimination in contracting.

3.7.2 Equity Constraints

Given that subsidies and affirmative action may not lead to equitable contract awards among similarly situated disadvantaged firms, I now consider an intervention that does: equity constraints. In this environment, an equity constraint specifies a win probability for the unestablished disadvantaged firm, $\gamma_U \in [0,1]$, and the established disadvantaged firm, $\gamma_E \in [0,1]$, such that $\gamma_U + \gamma_E \leq 1$. The buyer awards the project randomly using the prescribed win probabilities, analogous to a lottery.

The benefit of implementing this policy is that the government can ensure both diversity and equity in awards. A disadvantaged firm wins with probability $\gamma_U + \gamma_E$, and the government can either split awards within the disadvantaged group (by setting $\gamma_U = \gamma_E$) or select win probabilities equal to the proportion of unestablished and established disadvantaged firms in the market.[10]The cost of this policy is that the government has effectively removed all competition between disadvantaged firms and the outside option and replaced it with pre-specified win probabilities. Proposition 6 shows how this change affects investment in equilibrium.

Proposition 6. *Let $G(c)$ be continuous and satisfy $G(0) = 0$. Then for any equity constraint with $\gamma_U, \gamma_E \in [0,1]$ and $\gamma_U + \gamma_E \leq 1$, the unique equilibrium is $\pi_j^* = 0$ for each j.*

In words, Proposition 6 says that neither the established nor unestablished disadvantaged firm invests in any equilibrium when there are equity constraints, provided that investment costs are non-negative. The intuition is that investment no longer gives disadvantaged firms a leg up on their competition, as win probabilities are constrained to be fixed. Thus, investment incentives disappear, leading to a unique no-investment equilibrium. These results show that

[10]For example, if ten percent of firms are disadvantaged and unestablished and five percent of firms are disadvantaged and established, then the government can set $\gamma_U = 0.1$ and $\gamma_E = 0.05$.

even well-intentioned equity policies may produce undesirable outcomes in contracting if they disregard competition between firms.

4 Discussion

The model relies on several assumptions to generate a parsimonious analysis of equity and affirmative action in government contracting. However, one may question how robust the results are to alternative modeling choices on the number of competing contractors and group designations. In this section, I discuss some of these alternatives and how they would affect the analysis.

Adding an Advantaged Firm

The model abstracts away from choices made by advantaged firms by assuming they are part of the outside option. An alternative to this assumption would be to add a firm belonging to an advantaged group with the same investment cost distribution as the two disadvantaged firms while re-interpreting the outside option as an in-house cost only.

Most of the model's insights carry over with this change. Without affirmative action, one can construct symmetric equilibria, where the buyer treats all firms equally, and asymmetric equilibria, where there is some form of exclusion. Having three firms would allow for the exclusion of one or both disadvantaged firms, whereas the exclusion of both firms in the two-firm model corresponds to the symmetric equilibrium with no investment. When there is affirmative action, the problem for the disadvantaged firms is unchanged, as missing the threshold triggers an automatic loss to the outside option. The downside of the three-firm model is that it adds many new terms to the investment incentive function; firms there would need to account for all combinations of competitor investment decisions when evaluating their probability of winning. For that reason, I focus on the two-firm model.

Adding an Arbitrary Number of Firms

Related to the prospect of adding an advantaged firm is what would happen if, instead of one firm from each group, there were an arbitrary number of firms in each group. Given that competition between contractors is central to investment incentives, one might expect that some results are sensitive to the number of firms.

Aside from the investment-incentive expressions, which would need to account for all combinations of investment decisions between all competing firms, the propositions would still hold. Symmetric equilibria will still exist because the buyer can always choose to ignore labels, and asymmetric equilibria with exclusion (of all unestablished firms) will exist so long as the buyer believes no excluded firm will invest. The equality-constraint result also holds, provided that each firm has a valid win probability. The cumbersome expressions that come from having many firms motivate my focus on a single firm in each group.

Set-Asides

Consistent with how the government implements affirmative action in practice, the model has a posterior cutoff, $\overline{P}$, assumed to be positive. Consider now what would happen if $\overline{P} = 0$, so the buyer must use one of the two disadvantaged firms. This situation is analogous to a set-aside contract.

With these set-asides in place, one can show that any positive symmetric equilibrium is unique, as the investment-incentive line must be constant on its interior. The intuition is that the buyer sees both firms as equal under symmetry and will choose the high-signal firm—irrespective of their initial beliefs, $\pi \in (0, 1)$. At the endpoints, incentives are zero for the same reasons discussed earlier (investment would not change posteriors); thus, incentives can be discontinuous at the endpoints provided they are positive on the interior. As a result, there can be only one symmetric equilibrium with positive investment. Note that the no-investment equilibrium is still possible due to the left endpoint being zero.

Set-asides also change asymmetric equilibria with exclusion. Namely, there can be no asymmetries with exclusion with set-asides because the non-excluded firm's only competition is a non-investing disadvantaged firm. Because its only competitor does not invest, the non-excluded firm also has no incentive to invest. Note, however, that tie breaking leads to an outcome similar to exclusionary equilibria: in the symmetric no-investment equilibrium, neither disadvantaged firm invests, so the buyer always breaks ties in favor of the established firm. Thus, one can have outcomes consistent with exclusion even though the equilibrium is not technically asymmetric.

Investment Cost Asymmetry

The model assumes established and unestablished disadvantaged firms differ in labels only. In practice, however, firms may also have different investment cost distributions, which will change some of the model's implications.

As was the case in the symmetric equilibrium, the buyer can always choose to ignore labels. This choice will naturally lead to a higher investment probability for the cost-advantaged firm, coinciding with a higher equilibrium prior. Discrimination then becomes an issue, as the buyer may select an established disadvantaged firm with a higher prior over an unestablished firm with a higher signal. Subsidies can be a viable remedy to this issue. By decreasing the investment-cost gap through subsidization, firms become more "similar" to a label-ignoring buyer and are, therefore, more similarly used and less likely to face discrimination.

In contrast, the buyer can use labels in their decision-making process. That choice can lead to asymmetric equilibria with exclusion, with no investment reinforced by a zero prior. Moreover, it is possible for an overall less costly firm to be excluded, so a buyer's usage of labels can be problematic. In these instances, reversing discrimination requires a change in the buyer's priors reinforced by sufficient investment. In practice, networking programs—such as the Learning, Information, Networking, Collaboration initiative in Texas studied by De Silva et al. (2020)—may provide the conditions necessary to curtail discriminatory practices.

5 Conclusion

This paper investigates a contracting model where buyers can discriminate within the disadvantaged group of firms. Even when all disadvantaged firms have the same investment costs, there can be highly inequitable equilibria that arise when buyers consider cost-irrelevant labels, resulting in the disadvantaged firm with the unfavorable label never winning any contracts. These equilibria persist through affirmative action, although affirmative action may increase the probability that one of the disadvantaged firms wins the contract. A descriptive analysis from New Mexico confirms that such inequities can occur in practice. Furthermore, government interventions may not resolve these issues and may have the unintended consequence of discouraging investment.

Consistent with the type of discrimination reported by actual contractors, the labels I consider here are "established" and "unestablished." However, the analysis applies to any arbitrary labeling, so long as it falls within the disadvantaged category. Given that contracting agencies tend to use statistics from broader disadvantaged categories in evaluating their distributional objectives, they may not fully capture all forms of discrimination—undermining equity.

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A Proofs

A.1 Lemmas

I begin by establishing several lemmas to be used in the existence proofs.

Lemma 1. *$I_j(0) = \hat{I}_j(0; \overline{P}) = 0$ for $j \in \{E, U\}$.*

Proof. When $\pi = 0$, $P(\theta; 0) = 0$ for all θ.

When there is no affirmative action, note that $\overline{z} < p + x_u$ and $x_q < x_u$ by assumption, so $\frac{z-p-x_u}{x_q-x_u}$ is positive for every z and θ. Thus, $\Pr_j$(beat outside opt | invest) $=$ $\Pr_j$(beat outside opt | no invest) $= 0$ because $1\left\{P(\theta; 0) \geq \frac{z-p-x_u}{x_q-x_u}\right\} = 0$ for every θ and z. Hence, $I_j(0) = 0$.

When there is affirmative action, the assumption that $\overline{P} > 0$ implies $1\{P(\theta; 0) \geq \overline{P}\} = 0$ for all θ, so $\Pr_j(\text{win} \mid \text{invest}, \overline{P}) = \Pr_j(\text{win} \mid \text{no invest}, \overline{P}) = 0$. Hence, $\hat{I}_j(0; \overline{P}) = 0$. □

Lemma 2. *If $\lim_{\pi \nearrow 1} G(I_j(\pi)) < 1$ and π' is a point where $G(I_j(\pi')) > \pi'$, then there is another point $\pi'' \in (\pi', 1)$ such that $G(I_j(\pi'')) < \pi''$. Similarly, if $\lim_{\pi \nearrow 1} G(\hat{I}_j(\pi; \overline{P})) < 1$ and π' is a point where $G(\hat{I}_j(\pi'; \overline{P})) > \pi'$, then there is another point $\pi'' \in (\pi', 1)$ such that $G(\hat{I}_j(\pi''; \overline{P})) < \pi''$.*

Proof. I use contradiction. Suppose $\lim_{\pi \nearrow 1} G(I_j(\pi)) = L < 1$, but there is no $\pi'' \in (\pi', 1)$ where $G(I_j(\pi'')) < \pi''$. Then for every $\pi \in (\pi', 1)$, $G(I_j(\pi)) \geq \pi$. Take an ε-neighborhood around L small enough as to not contain 1 and its supremum—say $\varepsilon = \frac{1-L}{2}$ with supremum $\frac{L+1}{2}$. For any π close enough to 1, or in this case $\pi > \max\{\pi', \frac{L+1}{2}\}$, we would have $|G(I_j(\pi)) - L| \geq \varepsilon$, which contradicts $\lim_{\pi \nearrow 1} G(I_j(\pi)) = L$. The proof for $G(\hat{I}_j(\pi; \overline{P}))$ is the same and is thus omitted. □

Lemma 3. *$I_j(1) = \hat{I}_j(1; \overline{P}) = 0$ for $j \in \{E, U\}$.*

Proof. To prove this proposition, I show that the probability of winning given investment equals the probability of winning given no investment with and without affirmative action. Begin by noting that when $\pi = 1$, $P(\theta; 1) = 1$ for all θ and that the buyer breaks ties in favor of the established firm.

Without affirmative action, $\Pr_j$(beat outside opt | invest) $=$ $\Pr_j$(beat outside opt | no invest) $=$ 1 for both groups of firms because $1\left\{P(\theta; 1) \geq \frac{z-p-x_u}{x_q-x_u}\right\} = 1$ for all θ and z given the assumptions on z. For the established firm, $\Pr_E$(beat competitor | invest) $=$ $\Pr_E$(beat competitor | no invest) $= 1$ because $1\{P(\tilde{\theta}; 1) - P(\hat{\theta}; 1) \geq 0\} = 1$ for any $\tilde{\theta}$ and $\hat{\theta}$. Hence, $I_E(1) = 0$. For the unestablished firm, $\Pr_U$(beat competitor | invest) $=$ $\Pr_U$(beat competitor | no invest) $= 0$ because $1\{P(\tilde{\theta}; 1) - P(\hat{\theta}; 1) > 0\} = 0$ for any $\tilde{\theta}$ and $\hat{\theta}$. Thus, $I_U(1) = 0$.

Affirmative action changes the requirement that a disadvantaged firm beats the outside option in expectation to beating $\overline{P}$; the other probabilities mirror the case without affirmative action. Because $\overline{P} \leq 1$, $1\{P(\theta; 1) \geq \overline{P}\} = 1$ for all θ. Thus,

$$\int_0^1 1\{P(\tilde{\theta}; 1) \geq \overline{P}\} f_q(\tilde{\theta}) d\tilde{\theta} = \int_0^1 1\{P(\tilde{\theta}; 1) \geq \overline{P}\} f_u(\tilde{\theta}) d\tilde{\theta} = 1,$$

implying $\hat{I}_j(1;\overline{P}) = 0$ for both groups.

□

Lemma 4. *$I_j(0,\pi_{-j}) = \hat{I}_j(0,\pi_{-j};\overline{P}) = 0$ for all $\pi_{-j} \in (0,1]$ and $j \in \{E,U\}$.*

Proof. This lemma's proof follows immediately from the proof of Lemma 1, as

$$1\left\{P(\theta;0) \geq \frac{z-p-x_u}{x_q-x_u}\right\} = 1\{P(\theta;0) \geq \overline{P}\} = 0$$

for every θ and z. □

Lemma 5. *$I_j(1,0) = \hat{I}_j(1,0;\overline{P}) = 0$ for $j \in \{E,U\}$.*

Proof. As was true in Lemma 3, $1\left\{P(\theta;1) \geq \frac{z-p-x_u}{x_q-x_u}\right\} = 1$ for every θ and z, so $\Pr_j$(beat outside opt | invest) = $\Pr_j$(beat outside opt | no invest) = 1. Furthermore, note that

$$1\{P(\tilde{\theta};1) - P(\hat{\theta};0) \geq 0\} = 1\{P(\tilde{\theta};1) - P(\hat{\theta};0) > 0\} = 1,$$

so $\Pr_j$(beat competitor | invest) = $\Pr_j$(beat competitor | no invest) = 1 for *both* groups of firms. When taken together, these indicator values imply $\Pr_j$(win | invest) = $\Pr_j$(win | no invest), or $I_j(1,0) = 0$ for both firm groups.

When there is affirmative action, it is still the case that $1\{P(\theta;1) \geq \overline{P}\} = 1$ for all θ and the other probabilities are the same as the case without affirmative action, so $I_j(1,0;\overline{P}) = 0$. □

A.2 Proofs of Propositions and Corollaries

Proof of Proposition 1. From Lemma 1, we have that $I_j(0) = 0$. Because $G(0) = 0$, an equilibrium exists where $\pi^* = 0 = G(I_j(0))$.

Now suppose there is some $\pi \in (0,1)$ such that $G^{-1}(\pi) < I_j(\pi)$, or equivalently, $G(I_j(\pi)) > \pi$. By Lemma 2, there must be another point $\pi' \in (\pi,1)$ such that $G(I_j(\pi')) < \pi'$. From the intermediate value theorem, it follows that there is a $\pi^* \in (\pi,\pi')$ such that $\pi^* = G(I_j(\pi^*))$. □

Proof of Proposition 2. Lemma 4 shows that $I_j(0,\pi_{-j}) = 0$ for all $\pi_{-j} \in (0,1]$. Thus, $0 = G(I_j(0,\pi_{-j}))$ holds for the excluded group.

For the non-excluded group, suppose that there is a $\pi_j \in (0,1)$ such that $G^{-1}(\pi_j) < I_j(\pi_j,0)$, or equivalently, $\pi_j < G(I_j(\pi_j,0))$. From Lemma 5 we have that $G(I_j(1,0)) = 0$, so there exists a $\pi_j^* \in (\pi_j,1)$ such that $\pi_j^* = G(I_j(\pi_j^*,0))$ by the intermediate value theorem. □

Proof of Corollary 1. This corollary follows immediately from the Proof of Proposition 1 by replacing $I_j(\pi)$ in that proof with $\hat{I}_j(\pi;\overline{P})$. □

Proof of Corollary 2. This corollary follows immediately from the Proof of Proposition 2 by replacing $I_j(\pi_j, 0)$ in that proof with $\hat{I}_j(\pi_j, 0; \overline{P})$. $\square$

Proof of Proposition 3. Lemma 3 implies that $\pi^* \neq 1$, as $G(I_j(1)) = 0$. Because $I_j(\pi^*) < \hat{I}_j(\pi^*; \overline{P})$ by assumption, $G(I_j(\pi^*)) < G(\hat{I}_j(\pi^*; \overline{P}))$. Furthermore, equilibrium behavior implies $G(I_j(\pi^*)) = \pi^* < G(\hat{I}_j(\pi^*; \overline{P}))$. By Lemma 2, there must be another $\pi' \in (\pi^*, 1)$ such that $G(\hat{I}_j(\pi'; \overline{P})) < \pi'$. An application of the intermediate value theorem implies there is a $\pi^{**} \in (\pi^*, \pi')$ where $\pi^{**} = G(\hat{I}_j(\pi^{**}; \overline{P}))$. $\square$

Proof of Proposition 4. First consider the non-excluded firm. Similar to the Proof of Proposition 3, $I_j(\pi_j^*, 0) < \hat{I}_j(\pi_j^*, 0; \overline{P})$ implies $\pi_j^* < G(\hat{I}_j(\pi_j^*, 0; \overline{P}))$. Noting that $\hat{I}_j(1, 0; \overline{P}) = 0$ from Lemma 5 and that $G(0) = 0$, the intermediate value theorem implies that there is a $\pi_j^{**} \in (\pi_j^*, 1)$ such that $\pi_j^{**} = G(\hat{I}_j(\pi_j^{**}, 0; \overline{P}))$.

For the excluded firm, note that $I_j(0, \pi_{-j}) = I_j(0, \pi_{-j}; \overline{P}) = 0$ for all $\pi_{-j} \in (0, 1]$ from Lemma 4. Therefore, zero investment is an equilibrium: $0 = G(\hat{I}_j(0, \pi_{-j}^{**}; \overline{P}))$. $\square$

Proof of Proposition 5. Let $\hat{G}(c)$ be the distribution function induced by the subsidy; that is, $\hat{G}(c) = G(c(1 - \phi))$. For the excluded firm, note that $\hat{G}(0) = 0$, so $0 = \hat{G}(I_j(0, \pi_{-j}))$ holds. The equilibrium conditions for the non-excluded firm follow from the Proof of Proposition 2, so there exists a $\pi_j^* > 0$ satisfying $\pi_j^* = G(I_j(\pi_j^*, 0))$. $\square$

Proof of Proposition 6. Given the constraints, the probability of winning is γ_U for the unestablished firm and γ_E for the established firm. An unestablished firm will invest if

$$\gamma_U p \leq \gamma_U p - c$$

or

$$c \leq [\gamma_U - \gamma_U]p = 0.$$

Because $G(0) = 0$, almost no unestablished firm invests.

A similar calculation for the established firm yields $c \leq [\gamma_E - \gamma_E]p = 0$. Therefore, neither firm has an incentive to invest for almost every c, and $\pi_j^* = 0$ for each j. $\square$